Antenna Arrays

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Linear and Planar Arrays

- Arrays of Two Isotropic Sources
- Principles of Pattern Multiplication
- Linear Array of N Elements with Uniform Amplitude
  - Broadside
  - Ordinary Endfire
  - Increased Directivity Endfire Array (IDEA)
  - Scanning Array
- Linear Arrays with Non-Uniform Amplitude
- Planar Arrays

Array of Two Isotropic Point Sources

$$E = E_o e^{-j\beta r_1} + E_o e^{-j\beta r_2}$$

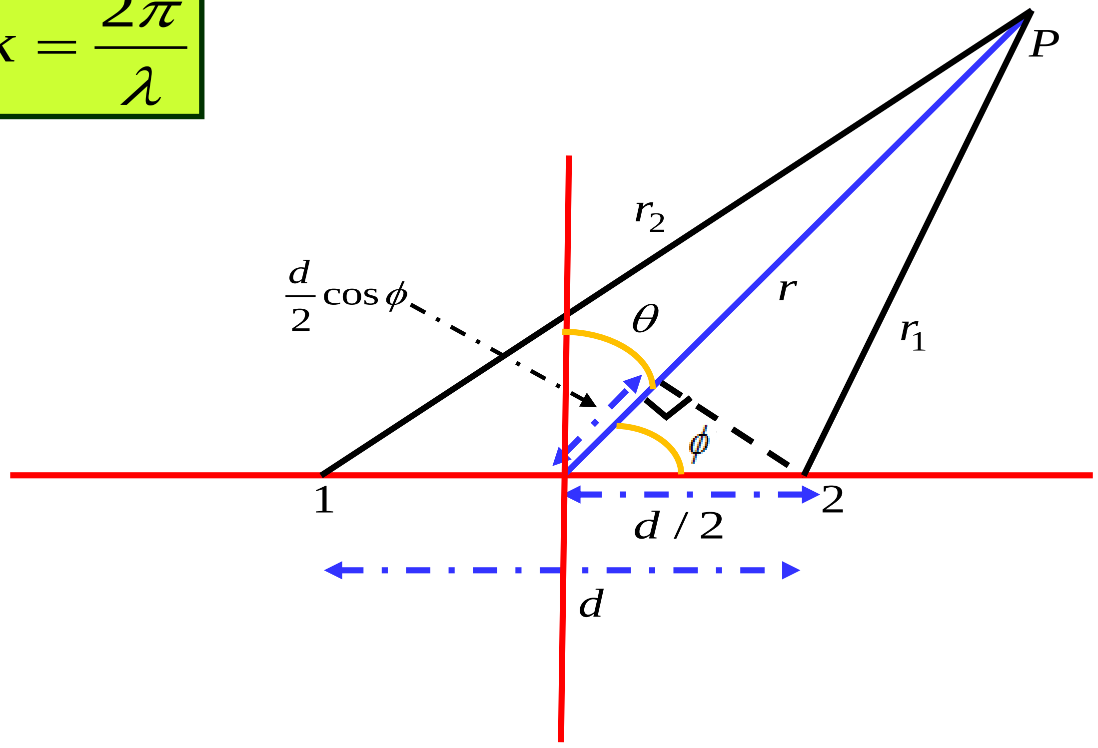
$$\beta = k = \frac{2\pi}{\lambda}$$

$$\left.\begin{aligned} r_1 &\cong r + \frac{d}{2}\cos\phi \\ r_2 &\cong r + \frac{d}{2}\cos\phi \end{aligned}\right) \quad r >> d, \phi = 90 - \theta$$

$$E = E_o e^{-j\beta r}\left[e^{-j\beta\frac{d}{2}\cos\phi} + e^{j\beta\frac{d}{2}\cos\phi}\right]$$

$$= E_o e^{-j\beta r}\left[e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right]$$

$$E = 2E_o \cos\left(\frac{\psi}{2}\right) = 2E_o \cos\left(\frac{\pi d}{\lambda}\cos\phi\right)$$



$$\psi = \beta d\cos\phi = \frac{2\pi d}{\lambda}\cos\phi$$

$$= \beta d\sin\theta = \frac{2\pi d}{\lambda}\sin\theta$$

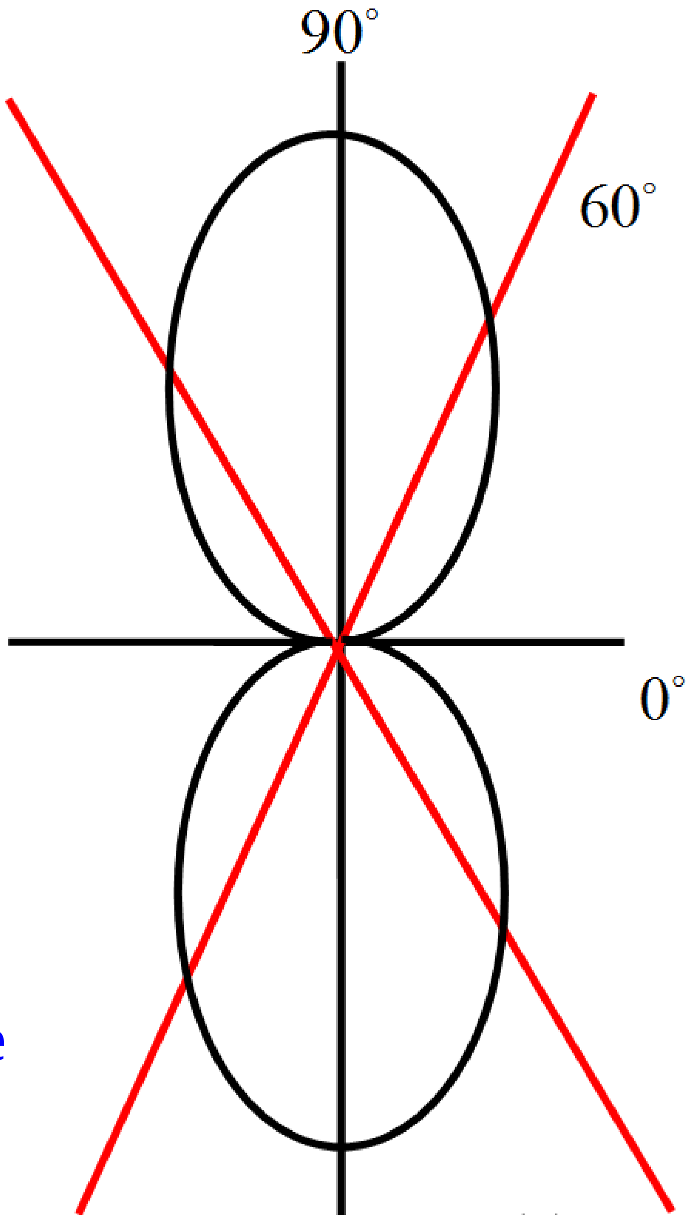
Two Isotropic Point Sources of Same Amplitude and Phase

$$E = \cos\left(\frac{d_r}{2}\cos\phi\right)$$

$$d_r = \frac{2\pi d}{\lambda} = \beta d$$

For $d = \frac{\lambda}{2}$ $\qquad E = \cos\left(\frac{\pi}{2}\cos\phi\right)$

ϕ	**0°**	**90°**	**60°**
E	**0**	**1**	$1/\sqrt{2}$



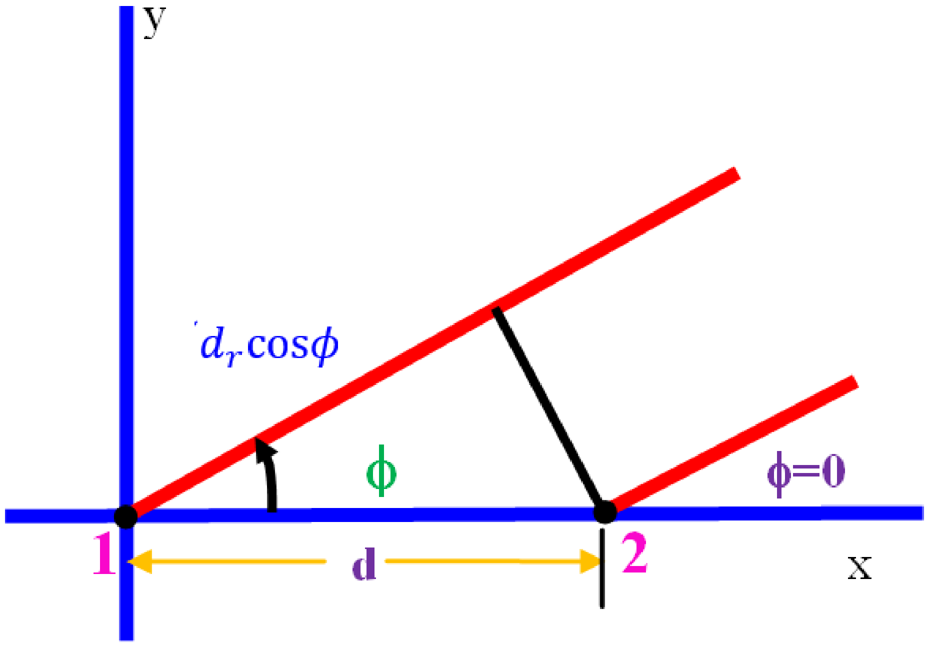
HPBWs = 60° in one plane and 360° in another plane

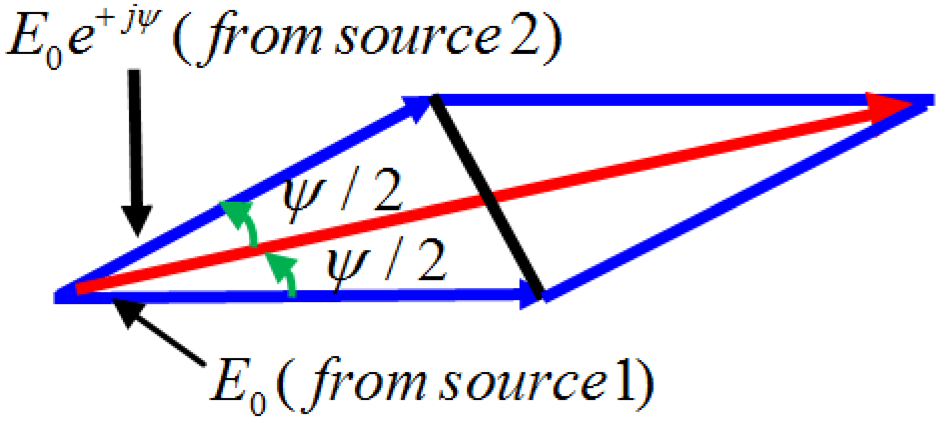
ORIGIN AT ELEMENT 1

$$E = E_0\left(1 + e^{j\psi}\right)$$
$$= 2E_0 e^{j\psi/2}\left(\frac{e^{j\psi/2}+e^{-j\psi/2}}{2}\right)$$
$$= 2E_0 e^{j\psi/2}\cos\frac{\psi}{2}$$

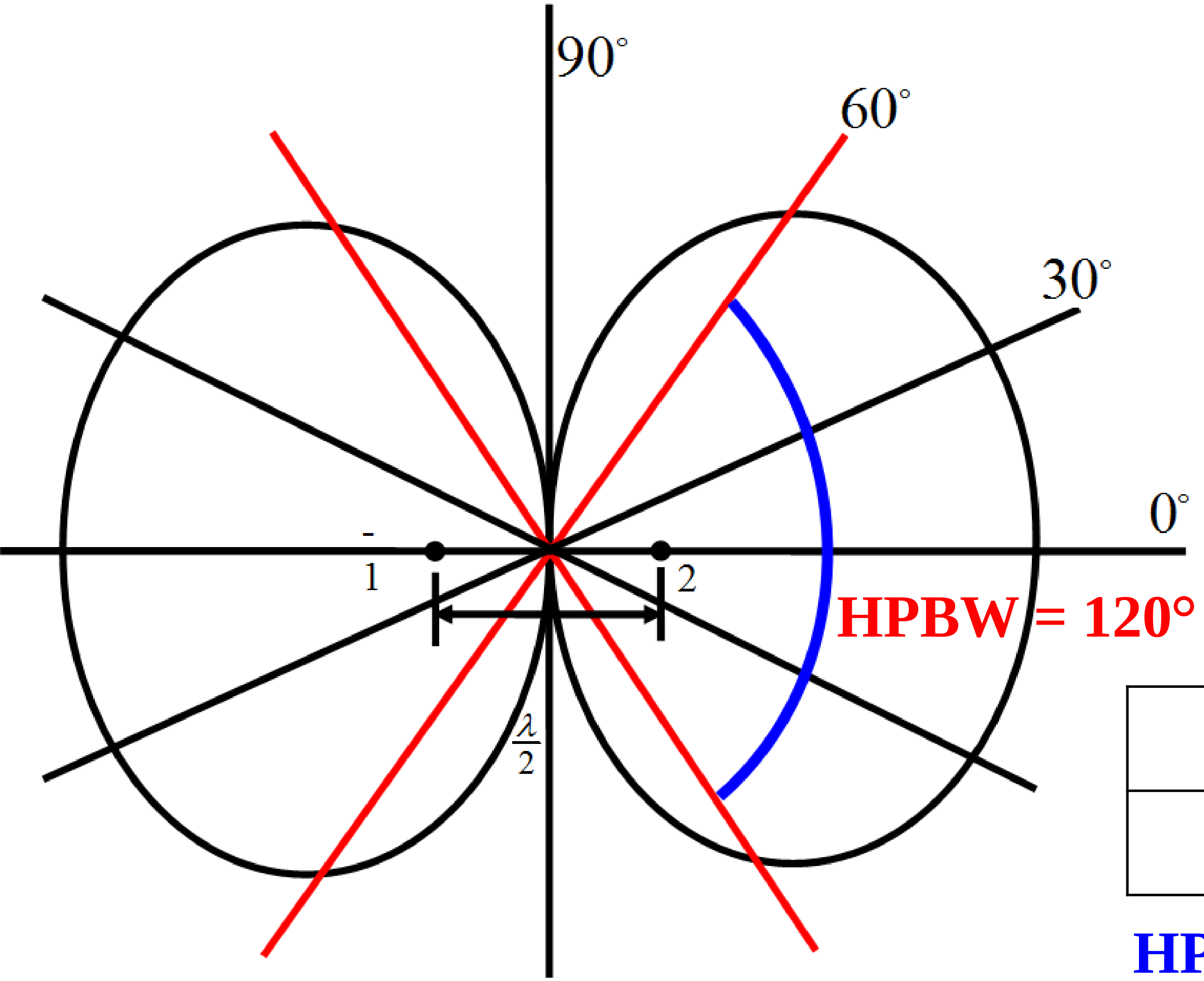
Normalizing by setting $2E_0 = 1$

$$E = e^{j\psi/2}\cos\frac{\psi}{2}$$
$$= \cos\frac{\psi}{2}\underline{|\psi/2}$$





Two Isotropic Point Sources of Same Amplitude and Opposite Phase



$$E = E_0 e^{+j\psi/2} - E_0 e^{-j\psi/2}$$

$$E = 2jE_0 \sin\frac{\psi}{2} = 2jE_0 \sin\left(\frac{d_r}{2}\cos\phi\right)$$

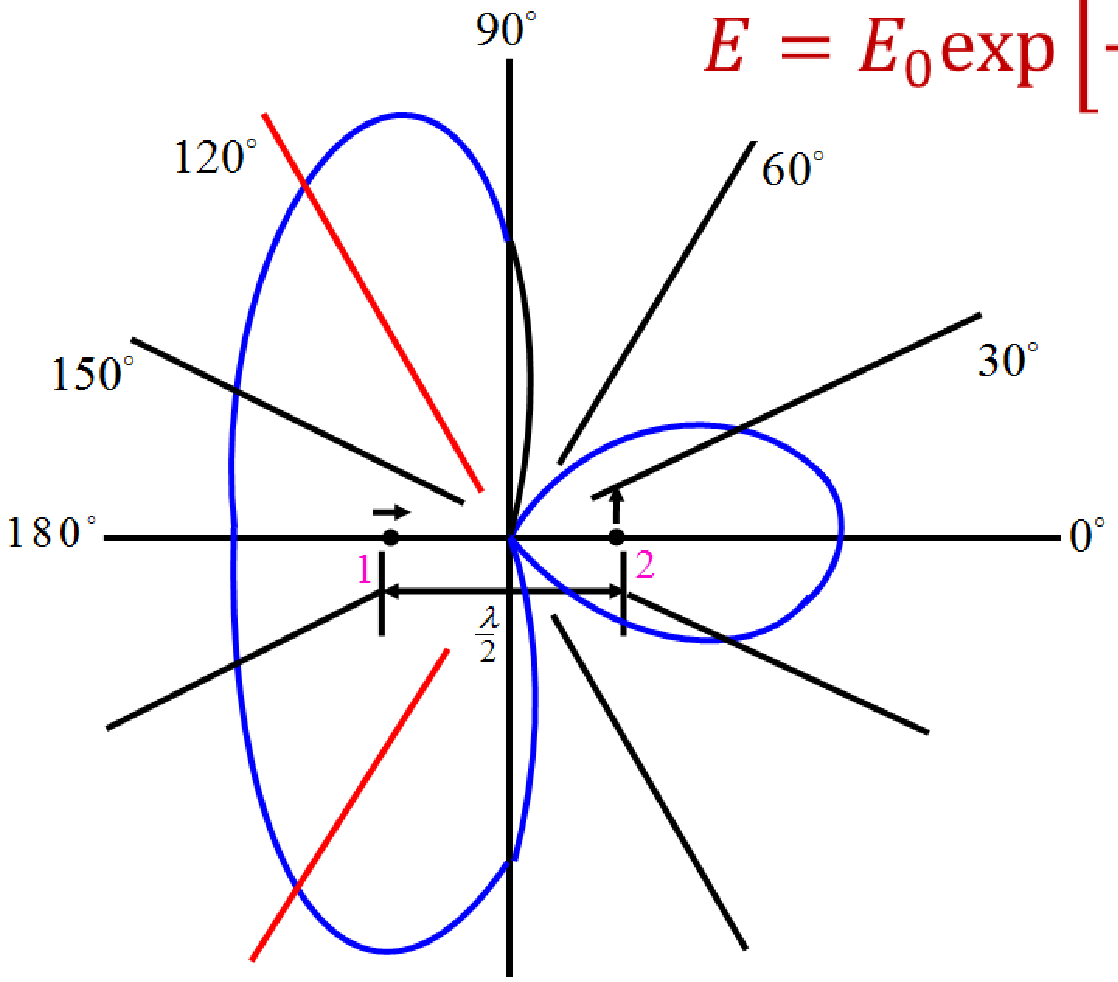
For $d = \frac{\lambda}{2}$

$$E = sin\left(\frac{\pi}{2}\cos\phi\right)$$

ϕ	0°	90°	60°
E	0	1	$1/\sqrt{2}$

HPBWs = 120° in both orthogonal planes

Two Isotropic Point Sources of Same Amplitude with 90^0 Phase Difference at λ/2



$$E = E_0 \exp\left[+j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right] + E_0 \exp\left[-j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right]$$

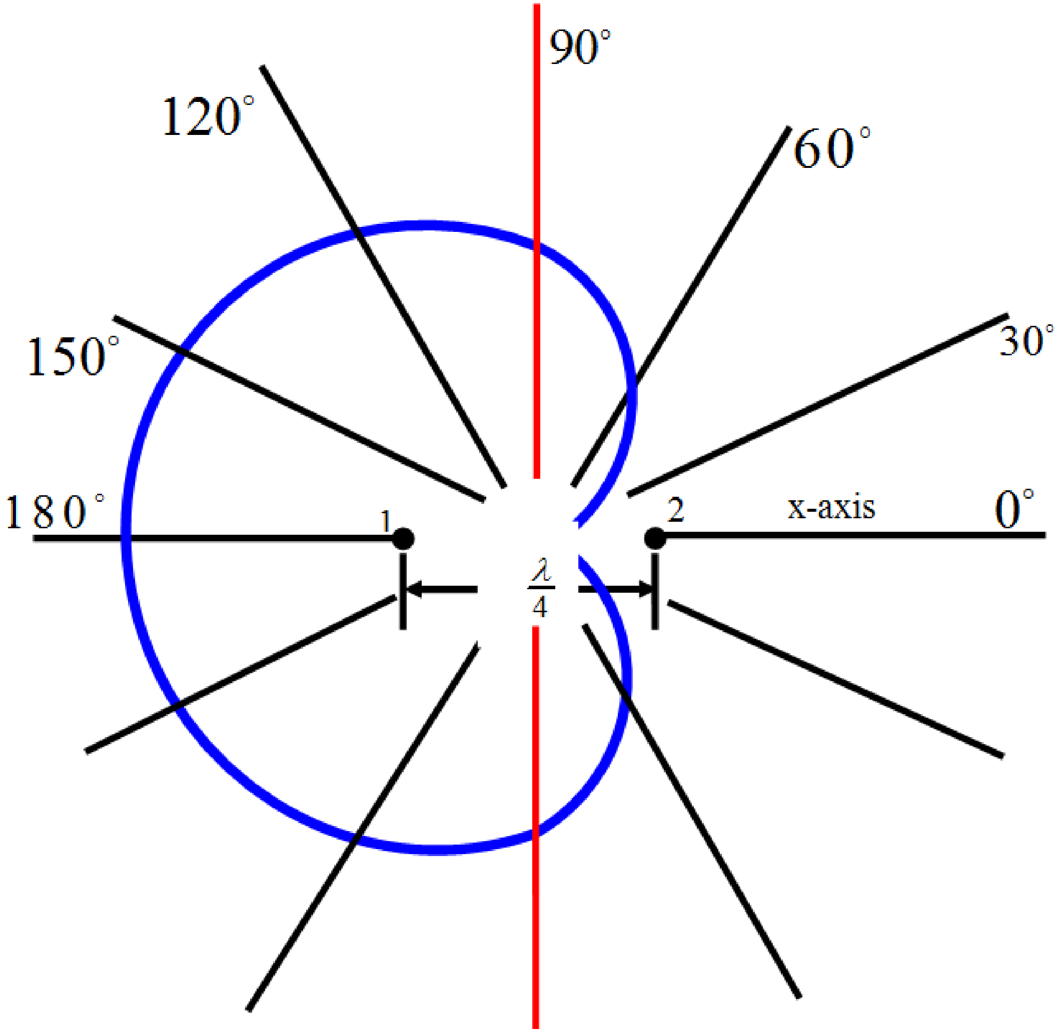
$$E = 2E_0 \cos\left(\frac{\pi}{4} + \frac{d_r}{2}\cos\phi\right)$$

Letting $2E_0 = 1$, and $d = \frac{\lambda}{2}$

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right)$$

ϕ	**0°**	**60°**	**90°**	**120°**	**180°**
E	$1/\sqrt{2}$	**0**	$1/\sqrt{2}$	**1**	$1/\sqrt{2}$

Two Isotropic Point Sources of Same Amplitude with 90^0 Phase Difference at λ/4



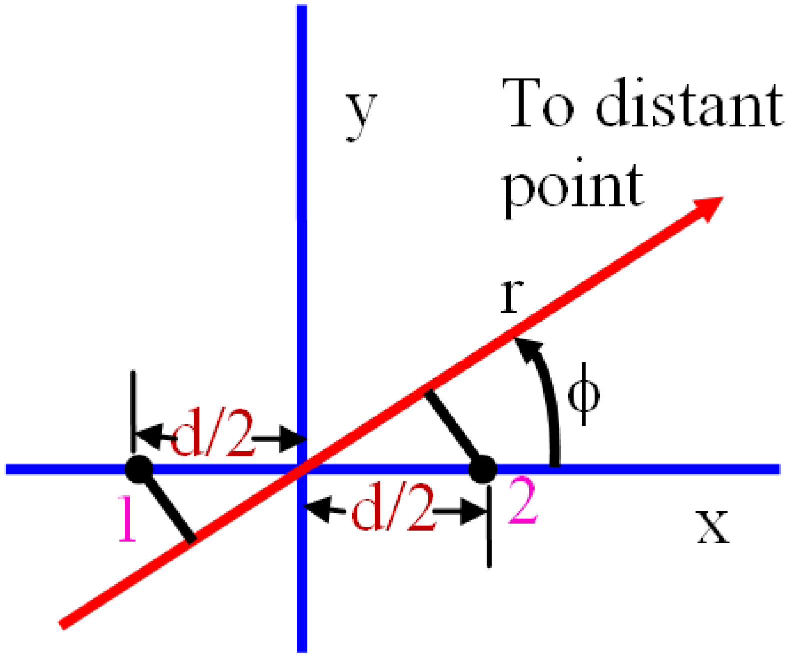
HPBW = 180°

Spacing between the sources is reduced to λ/4

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cos\phi\right)$$

ϕ	0°	90°	120°	150°	180°
E	0	$1/\sqrt{2}$	0.924	0.994	1

Two Isotropic Point Sources Of Same Amplitude with Any Phase Difference



$$\psi = d_r \cos\phi + \delta$$

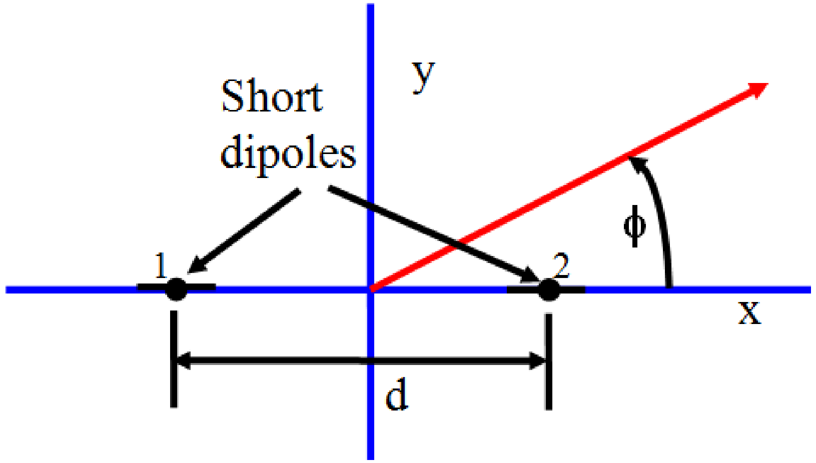
$$E = E_0\left(e^{j\psi/2} + e^{-j\psi/2}\right)$$

$$= 2E_0 \cos\frac{\psi}{2}$$

Normalizing by setting $2E_0 = 1$

$$E = \cos\frac{\psi}{2}$$

Two Same Dipoles and Pattern Multiplication

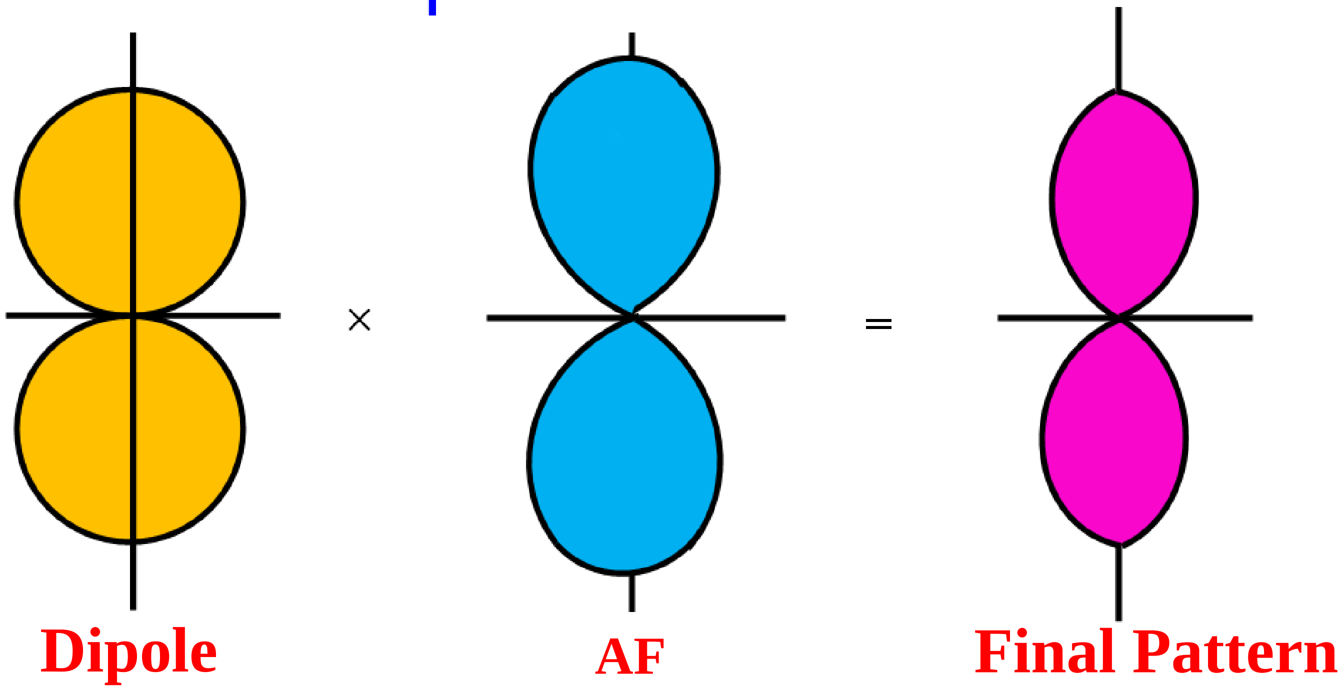


Dipole Pattern: $E_0 = E'_0 \sin\phi$

$$AF = \cos(\psi/2)$$

$$E = \sin\phi\cos\frac{\psi}{2}$$

where, $\psi = d_r\cos\phi + \delta$



For $\delta = 0$, Array Factor (AF) will give max. radiation in Broadside Direction

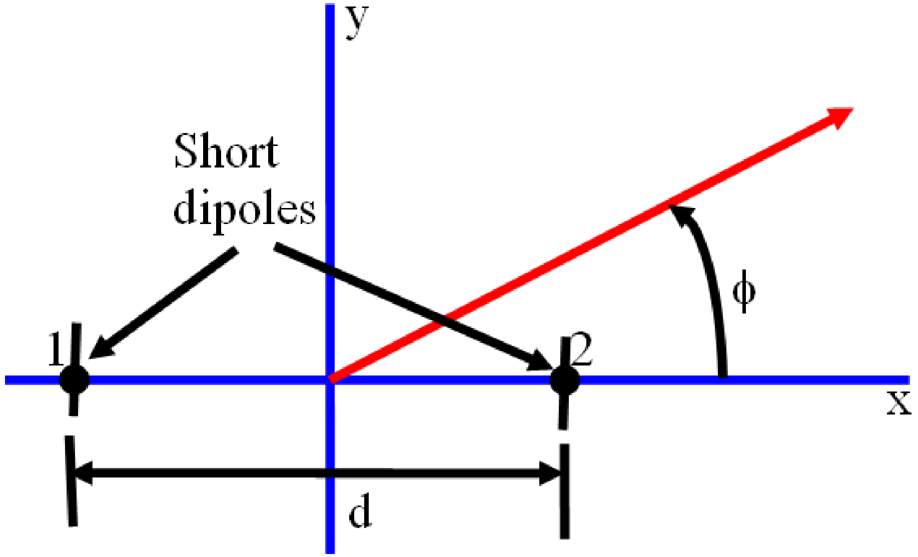
PATTERN MULTIPLICATION

Dipole E-Field for Vertical Orientation:

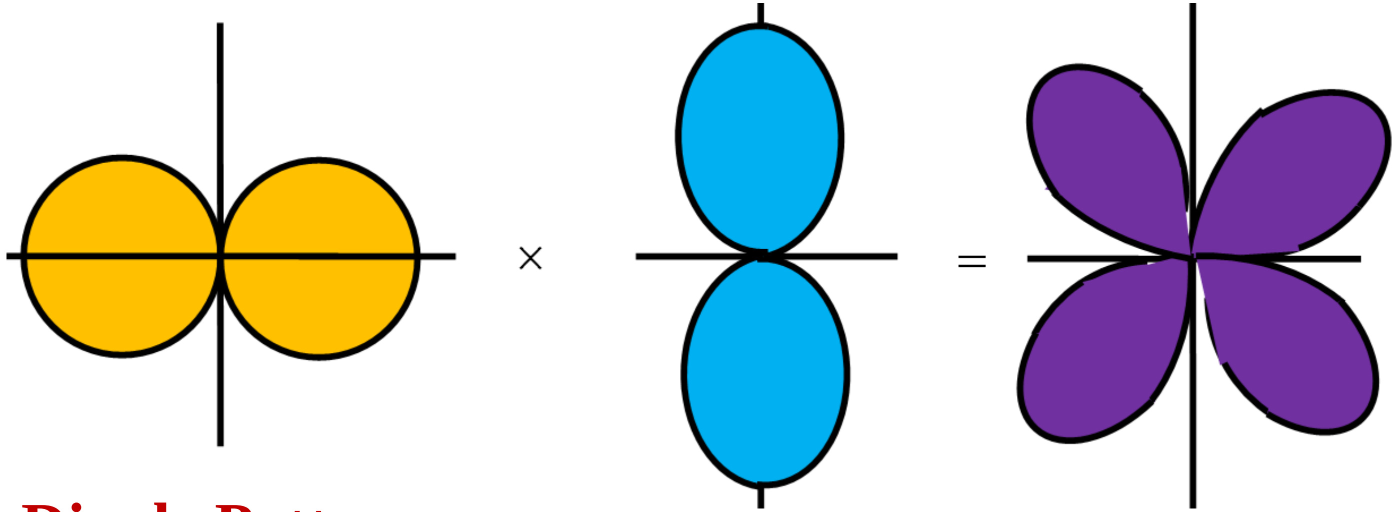
$$E_0 = E'_0 \cos\phi$$

Combined E-Field

$$E = \cos\phi \cos\left(\frac{\pi}{2}\cos\phi\right)$$

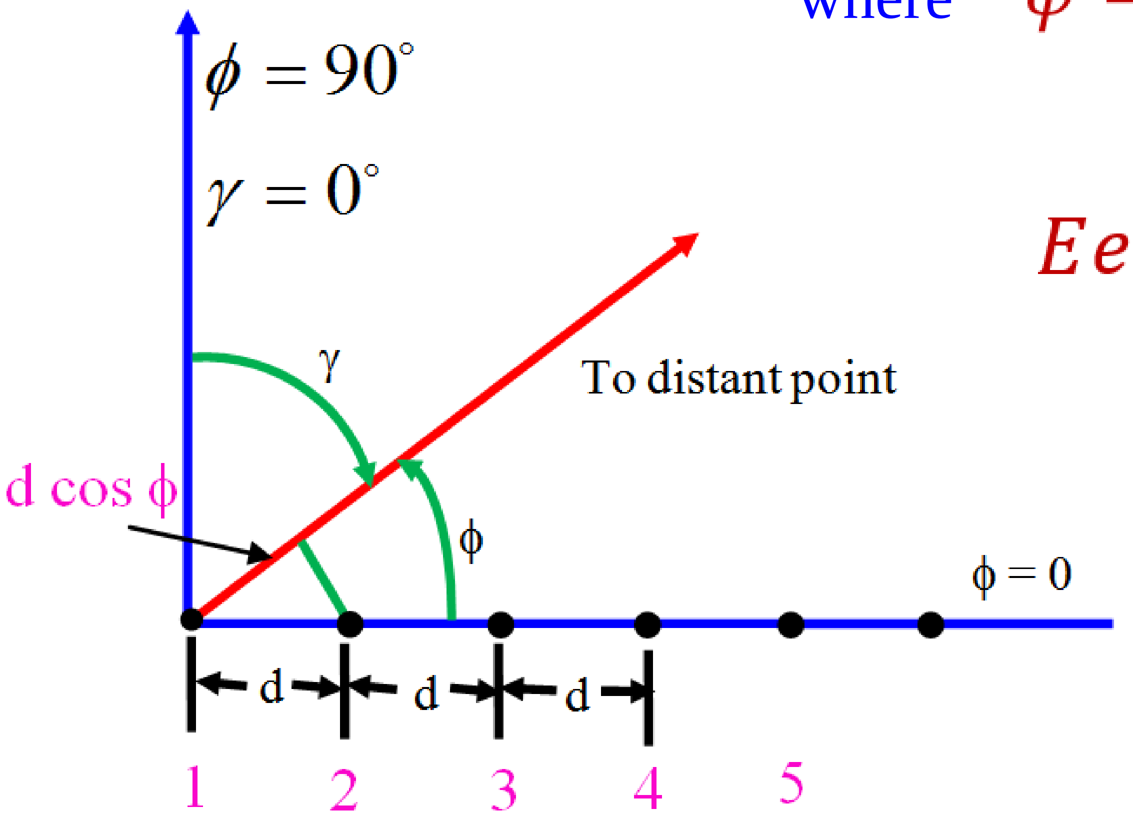


Array of two dipole antennas



Dipole Pattern **AF** **Product of Patterns**

N Isotropic Point Sources of Equal Amplitude and Spacing



$$E = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(n-1)\psi}$$

where $\psi = \dfrac{2\pi d}{\lambda}\cos\phi + \delta = d_r\cos\phi + \delta$

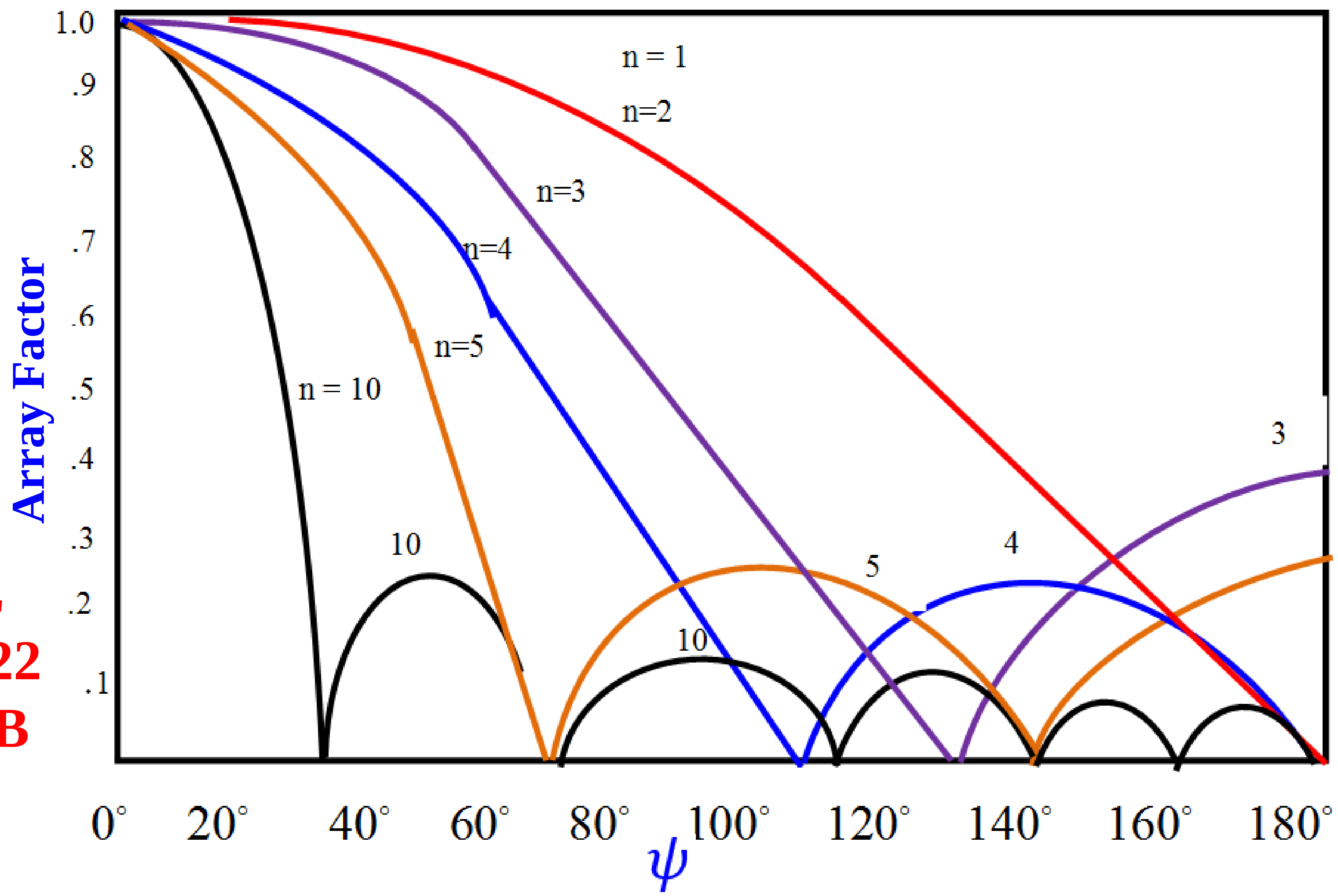
$$Ee^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{jn\psi}$$

$$E - Ee^{j\psi} = 1 - e^{jn\psi} = \frac{1 - e^{jn\psi}}{1 - e^{j\psi}}$$

$$E = e^{j\xi}\frac{\sin(n\psi/2)}{\sin(\psi/2)} = \frac{\sin(n\psi/2)}{\sin(\psi/2)}\angle\xi$$

$$E_{norm} = \frac{\sin(n\psi/2)}{n\sin(\psi/2)} \qquad \xi = \frac{n-1}{2}\psi$$

Radiation Pattern of N Isotropic Elements Array



First SLL
= 20log0.22
= -13.15dB

Radiation Pattern for array of n isotropic radiators of equal amplitude and spacing.

Broadside Array (Sources In Phase)

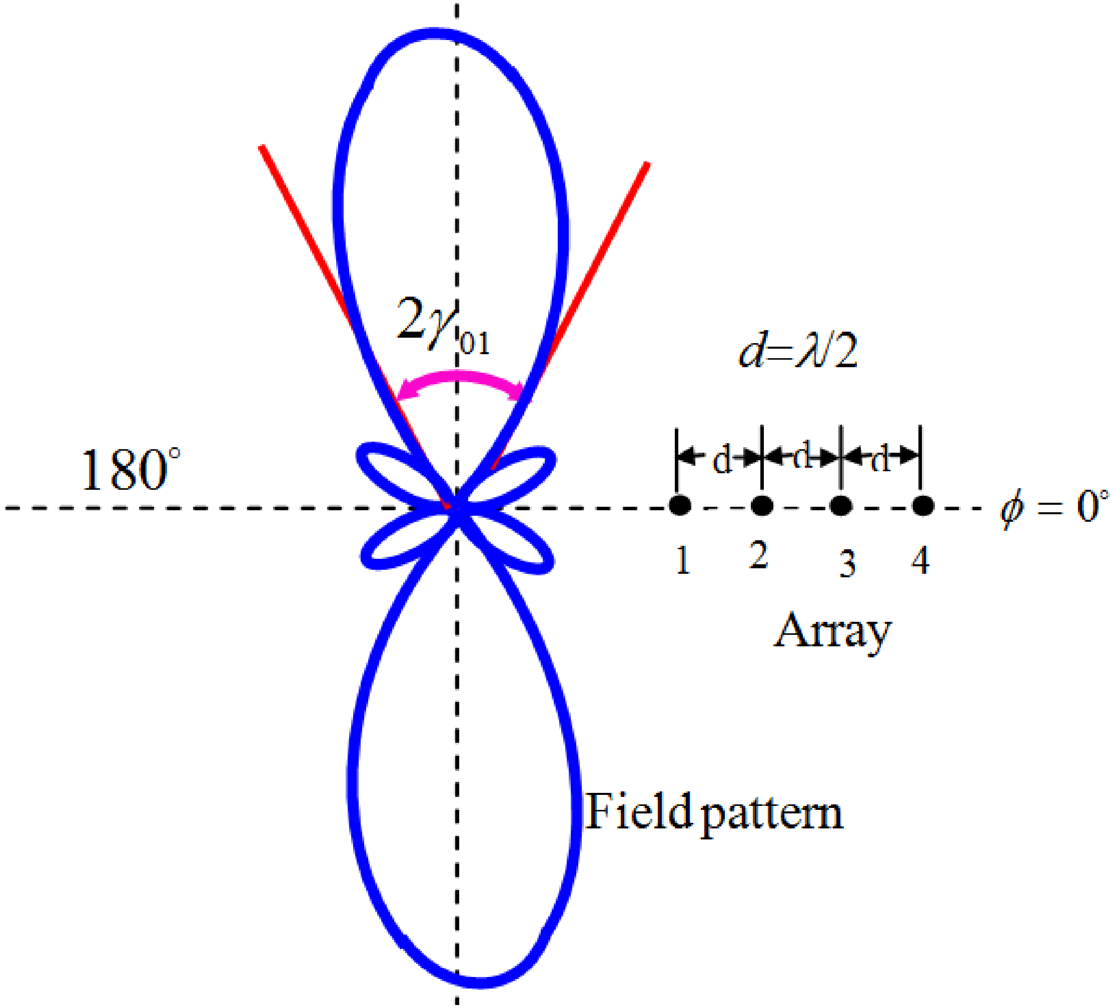
$$\psi = \frac{2\pi d}{\lambda}\cos\phi + \delta$$

$$\delta = 0,\ d = \frac{\lambda}{2}$$

$$\psi = \pi\cos\phi \qquad E = \frac{\sin(n\psi/2)}{n\sin(\psi/2)}$$

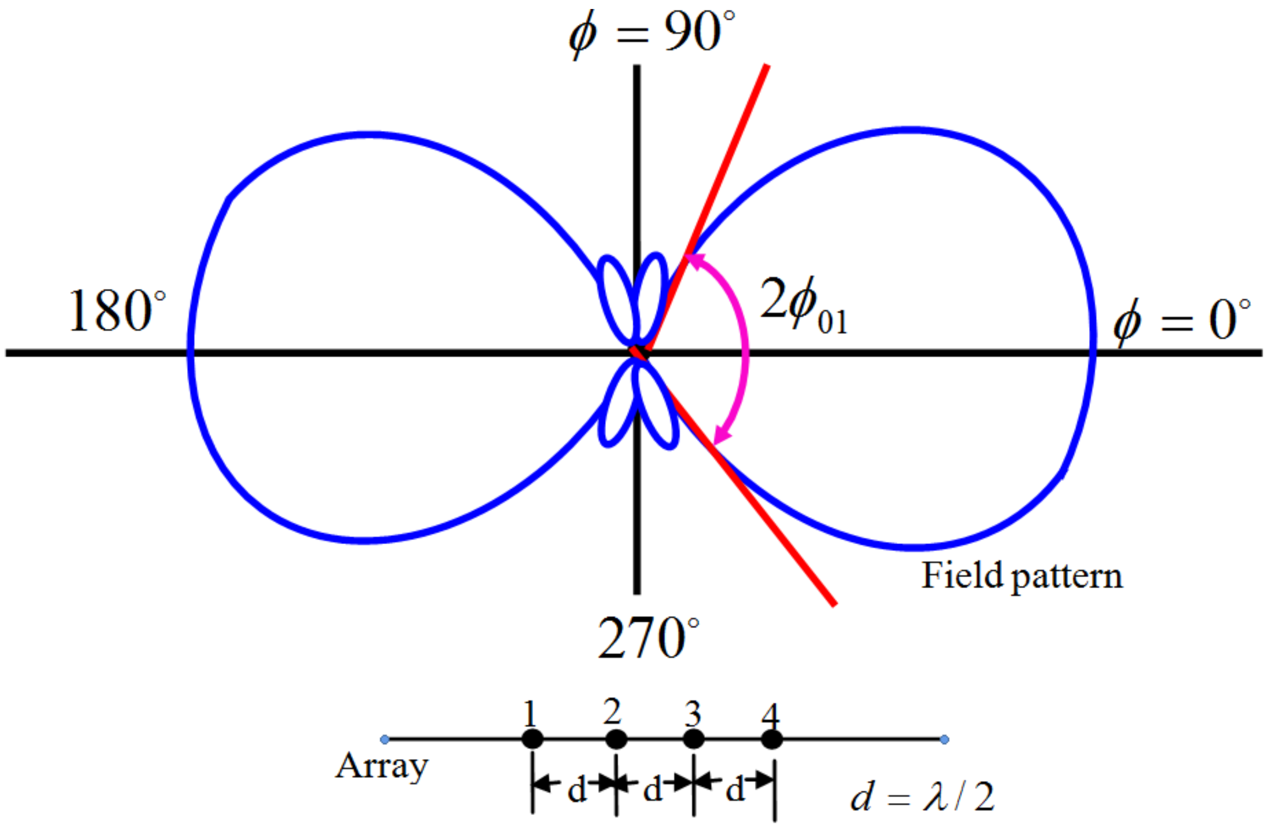
ϕ	Ψ	E
0°	π	0
90°	π/2	0
120°	0	1

$$BWFN = 2\gamma_{01} = 60°$$



Field pattern of 4 isotropic point sources with the same amplitude and phase and spacing of λ/2.

Ordinary Endfire Array



$\delta = -\pi$

$\psi = \pi(\cos\phi - 1)$

BWFN=120°

Field pattern of ordinary end-fire array of 4 isotropic point sources of same amplitude. Spacing is λ/2 and the phase angle δ = -π.

Increased Directivity Endfire Array (IDEA)

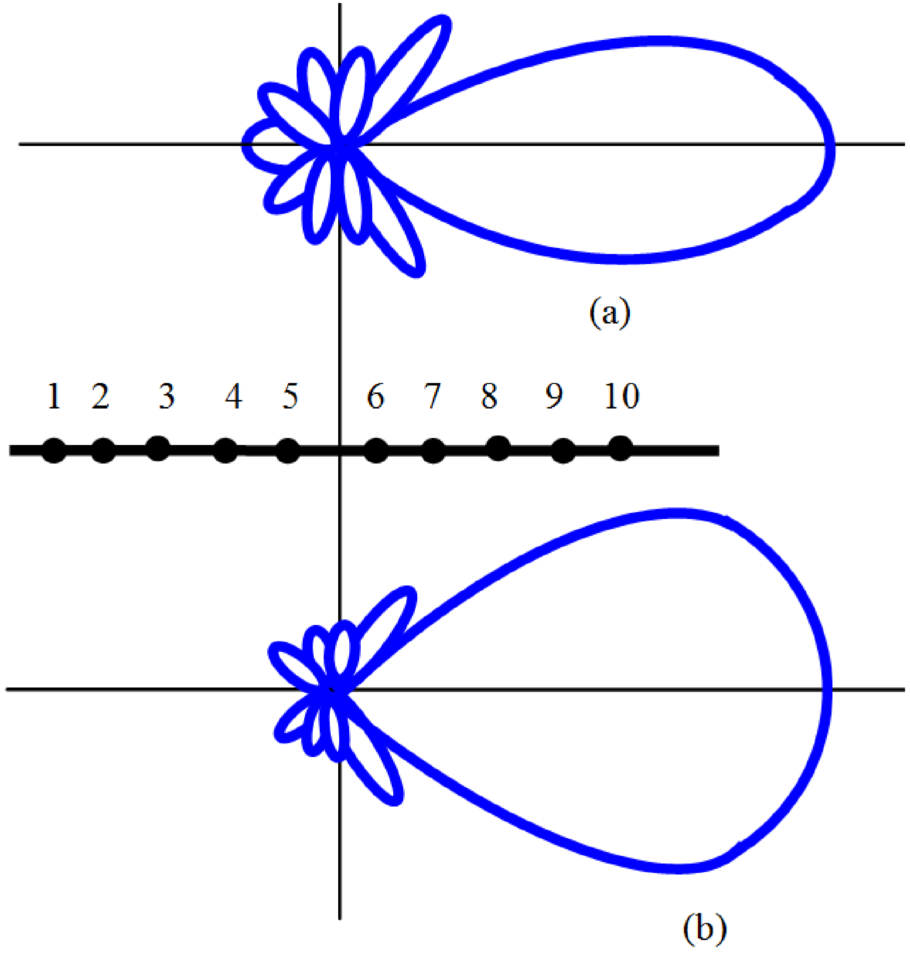
$$\psi = d_r(\cos\phi - 1) - \frac{\pi}{n}$$

Hansen and Woodyard criteria

$$\psi = d_r(\cos\phi - 1)$$

$$E_{norm} = \sin\left(\frac{\pi}{2n}\right)\frac{\sin(n\,\psi/2)}{\sin(\psi/2)}$$

Parameter	Ordinary end fire array	Endfire array with increased Directivity
HPBW	69°	38°
FNBW	106°	74°
Directivity	11	19



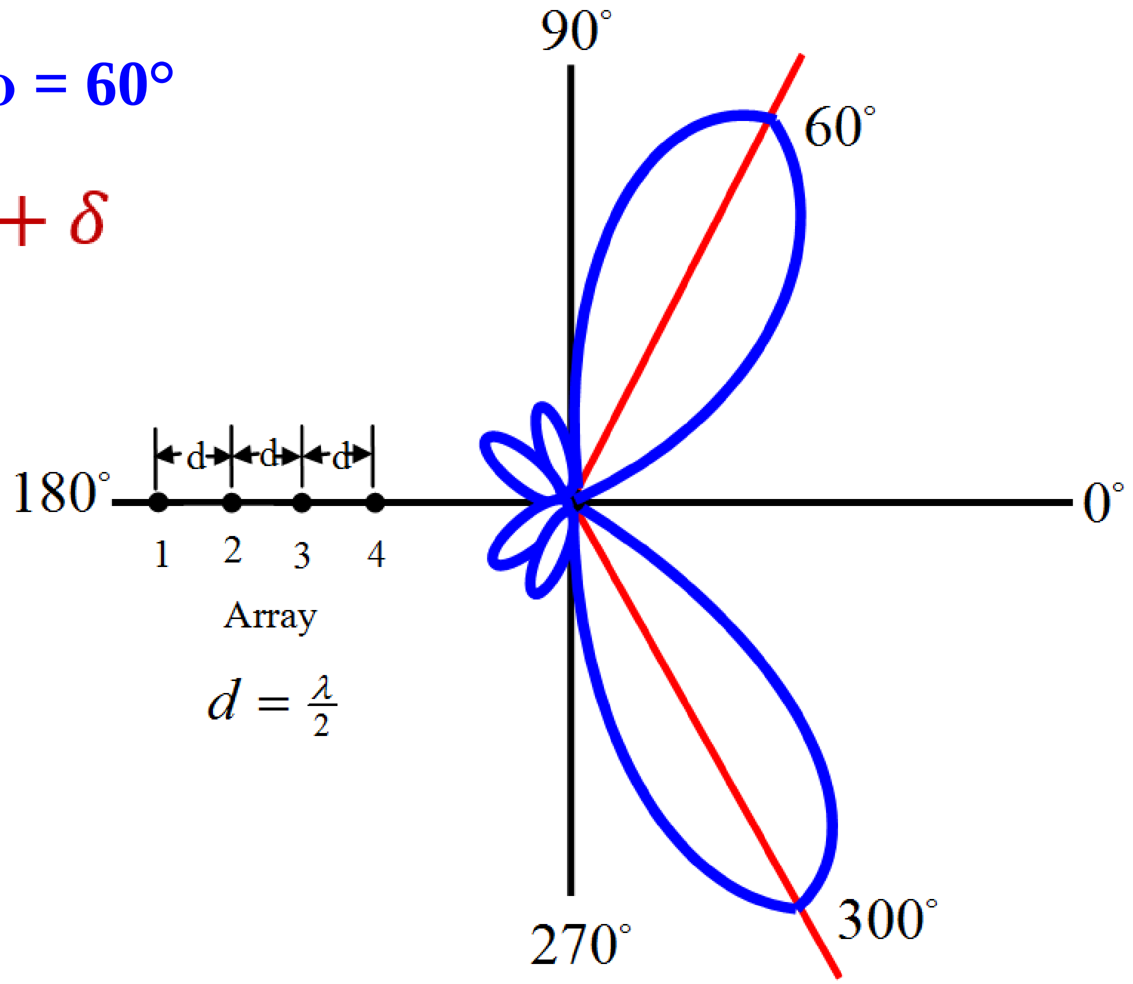
Field patterns of end-fire arrays of 10 isotropic point sources of equal amplitude spaced λ/4 apart.
(a) Phase for increased directivity (δ = -0.6π),
(b) Phase of an ordinary end-fire array (δ = -0.5π).

Array with Maximum Field in any Arbitrary Direction

For Beam Maxima at $\phi = 60°$

$$\psi = 0 = d_r \cos 60° + \delta$$

$$\delta = -\frac{\pi}{2}$$



Field pattern of array of 4 isotropic point sources of equal amplitude with phase adjusted to give the maximum at $\phi = 60°$ for spacing $d = \lambda/2$